

### ABSTRACT

The three-link mechanisms are the basis of most mechanical transmissions designed to convert the motion parameters of the input links. The general case diagram of such mechanism is examined, in order to define the possibility of obtaining three-link mechanisms able to perform the function of motion conversion at a high enough forward stroke efficiency in traction mode, and also, to ensure the mechanism self-stopping when necessary. The values of the back and forth stroke efficiency were obtained in a general form, and verified for screw-nut transmissions and screw and worm gears. The use of the obtained dependences during the mechanical transmissions design allows to determine more accurately the conditions for their operability.

**KEYWORDS:** Self-stopping mechanisms, Transmission screw-nut, Screw and worm gears, Mechanical transmissions theory.

### I. INTRODUCTION

The mechanical transmissions widely used in different kind of machines and engineering devices have a big variety of designs and they are produced in large quantities in mechanical engineering [1], [2]. The creation of general computing methods and design of mechanical transmissions allows us more accurately to determine the effect of the friction forces in the process of the power transmission from the input to the output link, to identify the possible modes of different types of mechanisms and to carry out the comparison and optimization of their results according to different criteria.

The one-degree-of-freedom 3-link mechanisms are the base of most of the mechanical transmissions designed to convert the motion parameters of the input links, or to convert one type of motion to another. These devices include the toothed and worm gears transmissions, screw-nut transmission, cam and wedge mechanisms, etc.

### Nomenclature

angle between the directions of the friction forces of the gears	$\delta_{12}$
angle between the directions of the friction forces of the gears	$\delta_{21}$
angle between the force direction $F_{n12}$ and axis of rotation of the link 1	$\gamma_{12}$
angle between the force direction $F_{n21}$ and axis of rotation of the link 2	$\gamma_{21}$
angle of the profile in the normal section	$\alpha_n$
angle of the thread profile in the transverse section	$\alpha_t$
angular velocity of the link 1	$\omega_1$
angular velocity of the link 2	$\omega_2$
arm of the force $F_{n20}$	$l$
arm of the force $F_{n21}$	$r_b$
arm of the force $F_{T21}$	$h$
arm of the friction force $F_{T12}$	$h_{12}$
arm of the friction force $F_{T21}$	$h_{21}$

arm of the normal force $F_{n12}$	$r_{12}$
arm of the normal force $F_{n21}$	$r_{21}$
arm of the resultant force $F_{n12}$	$r_{12}$
arm of the resultant force $F_{n21}$	$r_{21}$
axial component of the normal force $F_{n12}$	$F_{an12}$
axial component of the normal force $F_{n21}$	$F_{an21}$
coefficient of efficiency of the backward stroke	$\eta_{21}$
coefficient of efficiency of the forward stroke	$\eta_{12}$
component of the friction force $F_{T12}$	$F_{iT12}$
component of the friction force $F_{T21}$	$F_{iT21}$
external driving moment	$M_1$
external load moment	$M_2$
friction force	$F_{T20}$
friction force	$F_{aT2}$
friction moment	$M_{T1}$
friction moment	$M_{T2}$
generic coefficient of friction of the nut	$f_{21}^0$
helix angle of the teeth lines	$\beta_{w1}$
helix angle of the teeth lines	$\beta_{w2}$
initial circle	$r_{w1}$
initial circle	$r_{w2}$
lead angle of the screw thread	$\lambda$
lead angle of the worm thread in the initial circumference	$\lambda_q$
linear velocity	$v_1$
linear velocity	$v_2$
normal force	$F_{n20}$
number of starts of the nut thread	$z_2$
parameter of the friction links 1	$k_{12}$
parameter of the friction links 2	$k_{21}$
pitch of the nut thread	$P_X$
projection of the friction force $F_{T12}$ at the points of contact	$F_{iT12}$
projection of the friction force $F_{T21}$ at the points of contact	$F_{iT21}$
projection of the normal force $F_{n12}$ at the points of contact	$F_{m12}$
projection of the normal force $F_{n21}$ at the points of contact	$F_{m21}$
projection of the velocity $v_1$	$v_{n1}$
projection of the velocity $v_2$	$v_{n2}$
resultant normal force	$F_{n12}$
resultant normal force	$F_{n21}$
sum of the helix angles	$\sigma$

## II. MATERIALS AND METHODS

In order to define the possibility of obtaining three-link mechanisms able to perform the function of motion conversion maintaining a high enough forward stroke efficiency in traction mode, and also, to ensure the mechanism self-stopping when necessary, we examine the scheme of the general case of such mechanism designed to transmit the rotary motion between the intersecting axes (Fig. 1).

The diagram shows the loads on the mechanism links in traction mode of the forward stroke during the power transmission from the link 1 to the link 2. The backlash between the elements of the higher kinematic pair is provisionally shown.

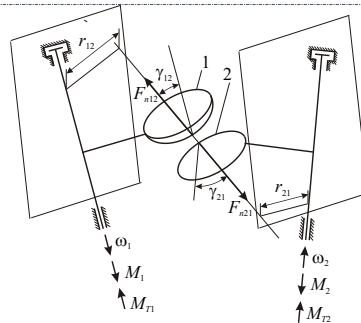


Fig. 1. Scheme of the three-link mechanism for the transmission of rotary motion between the intersecting axes

The driving input link 1 is loaded with the external driving moment  $M_1$ , which is led to the axis of this link and the driven output link 2 with the given external load moment  $M_2$ . The friction moments  $M_{T1}$  and  $M_{T2}$ , applied respectively to the links 1 and 2, consider all the friction forces, which arise at these links under the power transmission from the link 1 to the link 2.

The equilibrium equations of the links 1 and 2 at steady motion in traction mode of the forward stroke are given as:

$$M_1 = F_{n12}r_{12}\sin\gamma_{12} + M_{T1}; \quad (1)$$

$$M_2 = F_{n21}r_{21}\sin\gamma_{21} + M_{T2}, \quad (2)$$

Where  $F_{n12}$  and  $F_{n21}$  are the resultant normal forces on the kinematic pair elements, formed by the links 1 and 2;  $r_{12}$  and  $r_{21}$  are the arms of the resultant forces  $F_{n12}$  and  $F_{n21}$  corresponding to the rotation axes of the links 1 and 2 respectively;  $\gamma_{12}$  and  $\gamma_{21}$  are the angles between the forces direction action  $F_{n12}$  and  $F_{n21}$ , and the rotation axes of the links 1 and 2, respectively.

The equations (1) and (2) can be presented as:

$$M_1 = F_{n12}r_{12}\sin\gamma_{12}(1 + k_{12}); \quad (3)$$

$$M_2 = F_{n21}r_{21}\sin\gamma_{21}(1 + k_{21}), \quad (4)$$

where

$$k_{12} = \frac{M_{T1}}{F_{n12}r_{12}\sin\gamma_{12}}; \quad (5)$$

$$k_{21} = \frac{M_{T2}}{F_{n21}r_{21}\sin\gamma_{21}}. \quad (6)$$

In the equations (5) and (6), the values  $k_{12}$  and  $k_{21}$  are the friction parameters of the links 1 and 2. These values can be positive, if the direction of the resulting friction moment on the link coincides with the direction of the normal force moment corresponding to the rotation axis; and negative, if these directions are opposite.

The coefficient of efficiency of the forward stroke  $\eta_{12}$  of the mechanism in traction mode is defined as follows:

$$\eta_{12} = \frac{\omega_2 M_2}{\omega_1 M_1}, \quad (7)$$

Where  $\omega_1$  and  $\omega_2$  are angular the velocities of the links 1 and 2.

To determine the value of the angular velocities relation  $\omega_1/\omega_2$  in the equation (7), we find initially the linear velocities  $v_1$  and  $v_2$  of the points of application of the normal forces  $F_{n12}$  and  $F_{n21}$ :

$$v_1 = \omega_1 r_1; \quad v_2 = \omega_2 r_2. \quad (8)$$

The projections  $v_{n1}$  and  $v_{n2}$  of the velocities  $v_1$  and  $v_2$  for the action line of the forces  $F_{n12}$  и  $F_{n21}$ , i.e., for the direction of the general normals at the points of application of these forces are defined as follows:

$$v_{n1} = v_1 \sin\gamma_{12} = \omega_1 r_1 \sin\gamma_{12}; \quad (9)$$

$$v_{n2} = v_2 \sin\gamma_{21} = \omega_2 r_2 \sin\gamma_{21}. \quad (10)$$

According to the basic gearing theorem, these projections should be equal to:

$$v_{n1}, \text{ or } \omega_1 r_1 \sin\gamma_{12} = \omega_2 r_2 \sin\gamma_{21}, \quad (11)$$

from which

$$\frac{\omega_2}{\omega_1} = \frac{r_1 \sin \gamma_{12}}{r_2 \sin \gamma_{21}}. \quad (12)$$

Substituting the values  $M_1$  in the equation (7) and  $M_2$  from (3) and (4), the relation  $\omega_2/\omega_1$  from (12), and also considering that the modules of the forces  $F_{n12}$  and  $F_{n21}$  are equal, so we obtain after the conversion:

$$\eta_{12} = \frac{1+k_{21}}{1+k_{12}}. \quad (13)$$

If we analyse the general cases of transmission rotary to linear motion, linear to rotary motion or linear to linear motion by a three-link mechanism, it is easy to see that the general expression to determine the coefficient of efficiency is different from (13).

The coefficients  $k_{12}$  and  $k_{21}$  in the formula (13) for the links 1 and 2 of the three-link mechanism, which is assigned to convert linear to linear motion, will be identical to the friction parameters of the these units.

$$k_{12} = \frac{F_{aT1}}{F_{n12} \cos \alpha_1}; \quad (14)$$

$$k_{21} = \frac{F_{aT2}}{F_{n21} \cos \alpha_2} \quad (15)$$

Using the operability condition of the mechanism in traction mode of the forward stroke and the value of the friction parameters (14) and (15), we obtained the operability condition as  $k_{12} > k_{21} > -1$ ; and the condition of the mechanism self-stopping in traction mode of the forward stroke as  $k_{21} < -1$ .

The load scheme of the mechanism links in traction mode of the backward stroke differs from the scheme on the Figure 1 because the angular velocities of the links and the moments  $M_{T1}$  and  $M_{T2}$  of the friction forces have opposite directions. Consequently, the equation of the links motion will be:

$$M_2 = F_{n21} r_{21} \sin \gamma_{21} - M_{T2}; \quad (18)$$

$$M_1 = F_{n12} r_{12} \sin \gamma_{12} - M_{T1}. \quad (19)$$

If we use the parameters of the friction links in forward stroke, determined by the formulas (5) and (6) or (14) and (15), then the equations (18) and (19) can be presented as:

$$M_2 = F_{n21} r_{21} \sin \gamma_{21} (1 - k_{21}); \quad (20)$$

$$M_1 = F_{n12} r_{12} \sin \gamma_{12} (1 - k_{12}). \quad (21)$$

The coefficient of efficiency of the backward stroke  $\eta_{21}$  of the mechanism in traction mode will be equal to:

$$\eta_{21} = \frac{\omega_1 M_1}{\omega_2 M_2}. \quad (22)$$

Substituting in this equation the values of  $M_2$  and  $M_1$  from (20) and (21) with (12), we obtain:

$$\eta_{21} = \frac{1 - k_{12}}{1 - k_{21}}. \quad (23)$$

If the condition of the mechanism operability in traction mode of backward stroke (8) expressed in terms of the parameters of the friction links  $k_{12}$  and  $k_{21}$ , determined for the traction mode of the forward stroke (5) and (6) or (14) and (15), so it will given as:

$$1 \geq k_{12} > k_{21}. \quad (24)$$

According to (23) and (24), the self-stopping condition in traction mode of the backward stroke is given as:

$$k_{12} \geq 1. \quad (25)$$

To obtain the conditions of the mechanism operability, we examine the equilibrium equations of its links in this mode in Figure 1, for which the condition (25) is complete, i.e., to operate in release mode [1]. The scheme of the load on the mechanism links in brake release mode differs from the load scheme during the backward stroke only in that the moment  $M_1$  is a driving one, i.e. it has the same direction and angular velocity  $\omega_1$ , so the equation will be written as:

$$M_1 = -F_{n12} r_{12} \sin \gamma_{12} + M_{T1}; \quad (26)$$

$$M_2 = F_{n21} r_{21} \sin \gamma_{21} - M_{T2}, \quad (27)$$

or, by analogy with (20) and (21);

$$M_1 = F_{n12} r_{12} \sin \gamma_{12} (k_{12} - 1); \quad (28)$$

$$M_2 = F_{n21} r_{21} \sin \gamma_{21} (1 - k_{21}). \quad (29)$$

The release coefficient of the mechanism in brake release mode of the forward stroke according to [1] is:

$$\mu_{12} = \frac{\omega_1 M_1}{\omega_2 M_2} \tag{30}$$

After substituting the values  $M_1$  and  $M_2$  from (28) and (29), as well as the relations  $\frac{\omega_1}{\omega_2}$  from (12), the equation (30) is given as:

$$\mu_{12} = \frac{k_{12} - 1}{1 - k_{21}} \tag{31}$$

From the equation (31) it is known that the operability of the mechanism in brake release mode can be achieved only under the implementation of the conditions:

$$k_{21} < 1, \tag{32}$$

which, therefore, is the operability condition of the mechanism in this mode.

Using the obtained general formulas, we examine the examples of determining the efficiency and other characteristics, which depend on the friction forces for some three-link mechanisms.

**Screw-nut transmission**

Figure 2 shows the load scheme on the screw 1 and the nut 2 in traction mode of the forward stroke during the conversion of the rotary motion of the screw 1 to the linear motion of the nut 2.

The geometrical parameters of the screw and nut are assumed to be known.

All the normal forces on the contact surfaces of the screw and nut are conventionally led to a single point on the average diameter of the thread and replaced by the resultant normal forces  $F_{n12}$  on the screw and  $F_{n21}$  on the nut. Similarly, all the friction forces on these surfaces are replaced by the resultant friction forces  $F_{T12}$  and  $F_{T21}$  and applied on the same points. Figure 2 shows the axial components  $F_{an12}$  and  $F_{an21}$  of the normal forces  $F_{n12}$  and  $F_{n21}$ , the components  $F_{iT12}$  and  $F_{iT21}$  of the friction forces  $F_{T12}$  and  $F_{T21}$  and also the normal force  $F_{n20}$  and the friction force  $F_{T20}$ , acting on the nut.

The generic coefficient of friction of the nut  $f_{21}^0$ , which consider the nut friction in the guide screws in addition to the coefficient of sliding friction on the screw surface, is determined as follows:

$$f_{21}^0 = \frac{F_{aT2}}{F_{n21} \cos \delta_2} = \frac{F_{aT2}}{F_{n21} \sin \lambda},$$

where  $\lambda$  is the lead angle of the screw thread.

$$F_{aT2} = F_{T21} \sin \lambda + F_{T20} = f_{21} F_{n21} \sin \lambda + f_{20} F_{n20}.$$

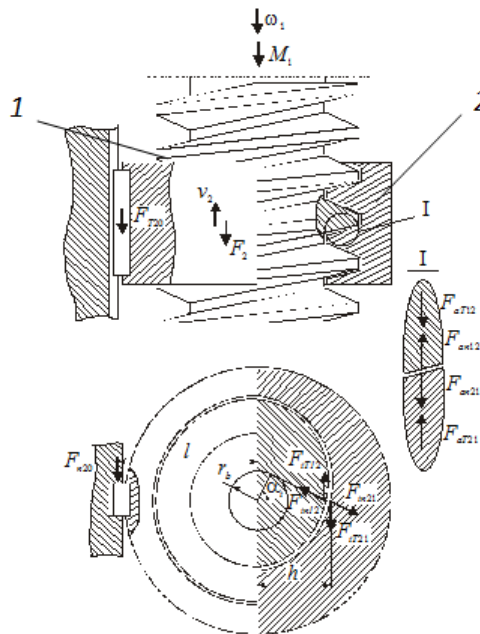


Figure 2 Scheme of forces in the screw-nut transmission

The normal force  $F_{n20}$  can be found from the equations of equilibrium of the nut relative to its axis:

$$F_{n20} = \frac{1}{l}(F_{n21}r_b + F_{T21}h) = \frac{F_{n21}}{l}(r_b \sin \gamma + f_{21}h \cos \lambda),$$

where  $l$ ,  $h$  and  $r_b$  are respectively the arms of the forces  $F_{n20}$ ,  $F_{T21}$ ,  $F_{n21}$  relative to the axis of the nut

$$r_b = \frac{P_x z_2}{2\pi \operatorname{tg} \gamma},$$

where  $P_x$  is the pitch of the nut thread;  $z_2$  is the number of starts of the nut thread.

Considering the found value of  $F_{n20}$ , the total projection of the friction force  $F_{aT2}$  will be equal to

$$F_{aT2} = F_{n21} \left[ f_{21} \sin \lambda + \frac{f_{20}}{l} (r_b \sin \gamma + f_{21} h \cos \lambda) \right],$$

and therefore, the generalized coefficient of friction  $f_{21}^0$  is determined as follows:

$$f_{21}^0 = f_{21} + \frac{f_{20}}{l} \left( r_b \frac{\sin \gamma}{\sin \lambda} + f_{21} h \operatorname{ctg} \lambda \right).$$

The parameter of friction of the screw  $k_{12}$  is given by the formula:

$$k_{12} = \frac{f_{12} h_2 \sin \delta_{12}}{r_b \sin \gamma},$$

where  $\delta_{12} = 90^\circ - \lambda$ , and the relation  $h_2/r_b$  can be expressed by the profile angle of the thread  $\alpha_n$  in the transverse section, which, in turn, depends on the angles  $\lambda$  and  $\gamma$  [2]:

$$\frac{r_b}{h_2} = \cos \alpha_n = \frac{\operatorname{tg} \lambda}{\operatorname{tg} \gamma}.$$

Also bearing in mind that the angles  $\gamma$ ,  $\lambda$  and  $\alpha_n$  are related by the correlation

$$\cos \gamma = \cos \lambda \cos \alpha_n,$$

the formula to determine the parameter of friction of the screw will be given as:

$$k_{12} = \frac{f_{12} \operatorname{tg} \gamma \cos \lambda}{\operatorname{tg} \lambda \sin \gamma} = \frac{f_{12} \cos \lambda}{\operatorname{tg} \lambda \cos \gamma} = \frac{f_{12}}{\cos \alpha_n} \operatorname{ctg} \lambda.$$

The parameter of friction of the nut will be:

$$k_{21} = -f_{21}^0 \frac{\cos \delta_{21}}{\cos \alpha_{21}} = -f_{21}^0 \frac{\sin \lambda}{\cos \gamma} = -\frac{f_{21}^0}{\cos \alpha_n} \operatorname{tg} \lambda.$$

The conditions of the non-self-stopping alternative of the screw-nut transmission, able to operate in traction mode of both forward and backward stroke, are as follows:

$$\operatorname{tg} \lambda > \frac{f_{12}}{\cos \alpha_n}; \operatorname{ctg} \lambda > \frac{f_{21}^0}{\cos \alpha_n} = \frac{1}{\cos \alpha_n} \left[ f_{21} + \frac{f_{20}}{l} \left( r_b \frac{\sin \gamma}{\sin \lambda} + f_{21} h \operatorname{ctg} \lambda \right) \right].$$

The coefficient of efficiency of the forward  $\eta_{12}$  and backward stroke  $\eta_{21}$  of this alternative is determined as follows:

$$\eta_{12} = \frac{1 - f_{21}^0 \frac{\operatorname{tg} \lambda}{\cos \alpha_n}}{1 + f_{12} \frac{\operatorname{ctg} \lambda}{\cos \alpha_n}} = \frac{1 - \left[ f_{21} + \frac{f_{20}}{l} \left( r_b \frac{\sin \gamma}{\sin \lambda} + f_{21} h \operatorname{ctg} \lambda \right) \right] \frac{\operatorname{tg} \lambda}{\cos \alpha_n}}{1 - f_{12} \frac{\operatorname{ctg} \lambda}{\cos \alpha_n}};$$

$$\eta_{21} = \frac{1 - f_{12} \frac{\operatorname{ctg} \lambda}{\cos \alpha_n}}{1 + f_{21}^0 \frac{\operatorname{tg} \lambda}{\cos \alpha_n}} = \frac{1 - f_{12} \frac{\operatorname{ctg} \lambda}{\cos \alpha_n}}{1 + \left[ f_{21} + \frac{f_{20}}{l} \left( r_b \frac{\sin \gamma}{\sin \lambda} + f_{21} h \operatorname{ctg} \lambda \right) \right] \frac{\operatorname{tg} \lambda}{\cos \alpha_n}}.$$

Without the calculation of the friction in the guide nut these formulas are as follows:

$$\eta_{12} = \frac{\operatorname{tg} \lambda}{\operatorname{tg}(\lambda + \rho)}; \eta_{21} = \frac{\operatorname{tg}(\lambda - \rho)}{\operatorname{tg} \lambda},$$

where

$$\rho = \operatorname{arctg} \frac{f_{12}}{\cos \alpha_n} = \operatorname{arctg} \frac{f_{21}^0}{\cos \alpha_n}.$$

The self-stopping performance of the screw-nut transmission, which has the properties of the first alternative of self-stopping, is possible under the conditions:

$$\operatorname{tg} \lambda < \frac{f_{12}}{\cos \alpha_n}; \operatorname{ctg} \lambda > \frac{f_{21}^0}{\cos \alpha_n} = \frac{1}{\cos \alpha_n} \left[ f_{12} + \frac{f_{20}}{l} \left( r_b \frac{\sin \gamma}{\sin \lambda} + f_{21} h \operatorname{ctg} \lambda \right) \right].$$

In this case, the coefficient of efficiency of the forward stroke is determined by the same formulas that we used for the non-self-stopping performance, but the brake release coefficient during the operation at release mode is determined with the formula

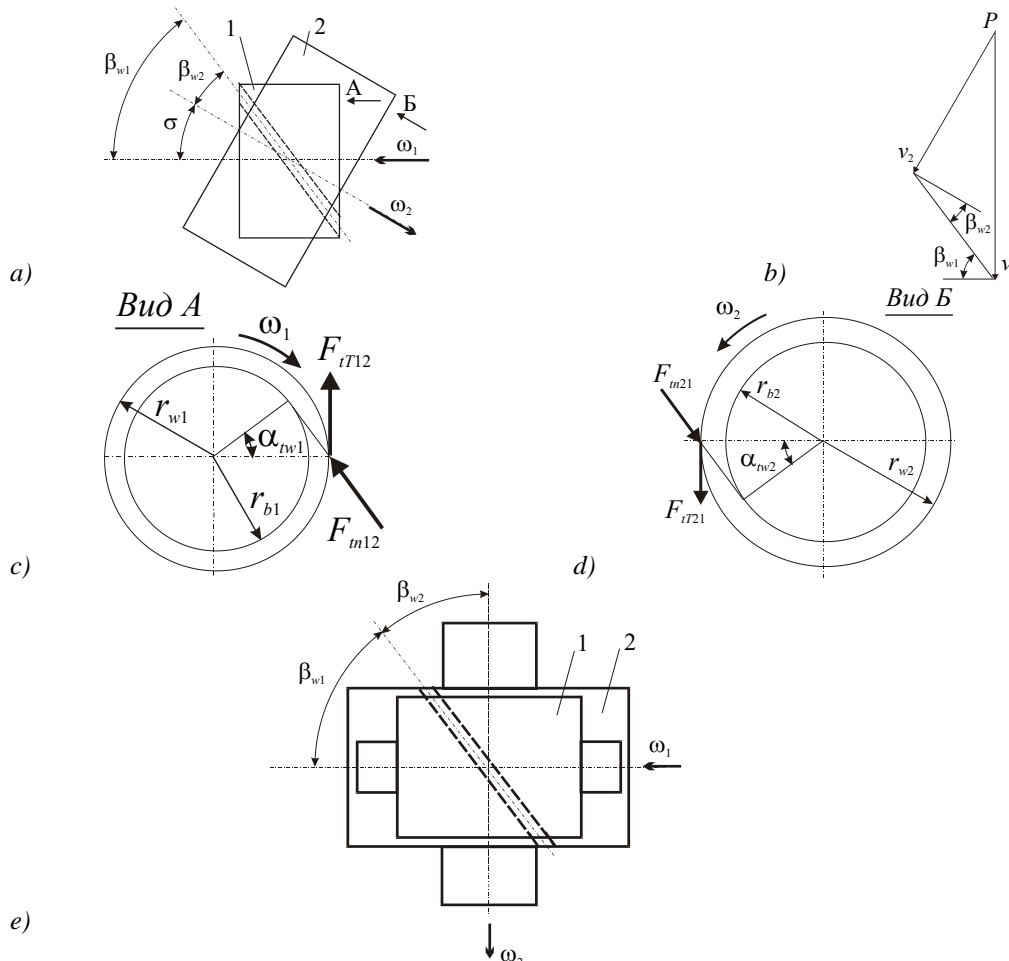
$$\mu_{12} = \frac{f_{12} \frac{\operatorname{ctg} \lambda}{\cos \alpha_n} - 1}{1 + \left[ f_{21} + \frac{f_{20}}{l} \left( r_b \frac{\sin \gamma}{\sin \lambda} + f_{21} h \operatorname{ctg} \lambda \right) \right] \frac{\operatorname{tg} \lambda}{\cos \alpha_n}},$$

or without calculate the friction in the guiding nut

$$\mu_{12} = \frac{\operatorname{tg}(\rho - \lambda)}{\operatorname{tg} \lambda}.$$

**Screw and worm gears**

The diagram of the hyperboloid screw transmission, formed with a zero involute cylindrical gears, is shown in Figure 3a. The diagram shows the first gear cylinders and the directions of the oblique teeth in the engagement. The screw lines of this teeth on the initial circumferences have the helix angles  $\beta_{w1}$  and  $\beta_{w2}$ , and the gears axes of in the diagram intersect in the space at an angle  $\sigma = \beta_{w1} - \beta_{w2}$ .



**Fig. 3. Hyperboloid screw transmission:**  
 a - diagram; b – plan of velocities; c – transverse section of the link 1;  
 d – transverse section of the link 2; e – diagram of the worm gear

Figure 3b shows the plan of velocities of the contact points, which determines the direction of their relative velocity and thus the direction of the friction forces at these points.

Figure 3c and d represent the projections  $F_{m12}$  and  $F_{m2}$  of the normal forces  $F_{n12}$  and  $F_{n21}$  at the points of contact and the projection  $F_{iT12}$  and  $F_{iT21}$  of the friction forces  $F_{T12}$  and  $F_{T21}$  at these points.

Figure 3e shows the diagram of the worm gear, which is a particular case of the screw hyperboloid transmission, when the sum of the helix angles of the lines of the gear teeth is  $90^\circ$ :

$$\sigma = \beta_{w1} + \beta_{w2} = 90^\circ.$$

The friction parameters of screw transmission gears in the Figure 3a without calculate the losses in the support are determined as:

$$k_{12} = f \frac{h_{12} \sin \delta_{12}}{r_{12} \sin \gamma_{12}}; \quad k_{21} = f \frac{h_{21} \sin \delta_{21}}{r_{21} \sin \gamma_{21}}.$$

In the diagrams of the Figure 3c and d, it is clear that the arms  $h_{12}$  and  $h_{21}$  of the friction forces  $F_{T12}$  and  $F_{T21}$  are equal according to the radii of the initial circumferences  $r_{w1}$  and  $r_{w2}$ , and the arms  $r_{12}$  and  $r_{21}$  of the normal forces  $F_{n12}$  and  $F_{n21}$  are equal to the main radii of the circumferences.

The angles  $\delta_{12}$  and  $\delta_{21}$  between the directions of the friction forces of the gears match to the helix angles  $\beta_{w1}$  and  $\beta_{w2}$  of the teeth lines. Thus, the formulas to determine the friction parameters of the gears can be written as:

$$k_{12} = f \frac{r_{w1} \sin \beta_{w1}}{r_{b1} \sin \gamma_{12}}; \quad k_{21} = f \frac{r_{w2} \sin \beta_{w2}}{r_{b2} \sin \gamma_{21}}.$$

Considering the correlations

$$\frac{r_{b1}}{r_{w1}} = \cos \alpha_{nw1} = \frac{1}{\operatorname{tg} \beta_{w1} \operatorname{tg} \gamma_{12}}; \quad \frac{r_{b2}}{r_{w2}} = \cos \alpha_{nw2} = \frac{1}{\operatorname{tg} \beta_{w2} \operatorname{tg} \gamma_{21}};$$

$$\cos \gamma_{12} = \sin \beta_{w1} \cos \alpha_n; \quad \cos \gamma_{21} = \sin \beta_{w2} \cos \alpha_n,$$

the formulae to determine  $k_{12}$  and  $k_{21}$  after the conversions are given as:

$$k_{12} = f \frac{\operatorname{tg} \beta_{w1}}{\cos \alpha_n}; \quad k_{21} = f \frac{\operatorname{tg} \beta_{w2}}{\cos \alpha_n},$$

Where  $\alpha_n$  is the angle of the profile in the normal section.

The obtained values of the gears friction can only be used for transmissions similar to that shown in Figure 3a, where the crossed axes angle  $\delta$  is determined by the difference  $\beta_{w1}$  and  $\beta_{w2}$ . If the angle  $\delta$  is equal to their sum, then only the parameter  $k_{12}$  will be positive, and the parameter  $k_{21}$  will be negative.

For example, for a worm gear, where  $\operatorname{tg} \beta_{w2} = \operatorname{ctg} \beta_{w1}$ , these parameters will be equal:

$$k_{12} = f \frac{\operatorname{tg} \beta_{w1}}{\cos \alpha_n}; \quad k_{21} = -f \frac{\operatorname{ctg} \beta_{w1}}{\cos \alpha_n}.$$

According to the condition (36), the screw transmission will be able to operate in traction mode of both forward and backward stroke, i.e. it will be non-self-stopping if:

$$\operatorname{ctg} \beta_{w1} < \frac{f}{\cos \alpha_n}; \quad \operatorname{ctg} \beta_{w2} < \frac{f}{\cos \alpha_n}.$$

The coefficient of efficiency of the forward stroke  $\eta_{12}$  and backward stroke  $\eta_{21}$  in this case is determined:

- at  $\sigma = \beta_{w1} - \beta_{w2}$ :

$$\eta_{12} = \frac{\cos \alpha_n + f \operatorname{tg} \beta_{w2}}{\cos \alpha_n + f \operatorname{tg} \beta_{w1}}; \quad \eta_{21} = \frac{\cos \alpha_n - f \operatorname{tg} \beta_{w1}}{\cos \alpha_n - f \operatorname{tg} \beta_{w2}};$$

- at  $\sigma = \beta_{w1} + \beta_{w2}$ :

$$\eta_{12} = \frac{\cos \alpha_n - f \operatorname{tg} \beta_{w2}}{\cos \alpha_n + f \operatorname{tg} \beta_{w1}}; \quad \eta_{21} = \frac{\cos \alpha_n - f \operatorname{tg} \beta_{w1}}{\cos \alpha_n + f \operatorname{tg} \beta_{w2}};$$

- worm gears:

$$\eta_{12} = \frac{\operatorname{tg}(\beta_{w1} - \rho)}{\operatorname{tg} \beta_{w1}} = \frac{\operatorname{tg} \lambda_4}{\operatorname{tg}(\lambda_4 + \rho)}; \quad \eta_{21} = \frac{\operatorname{tg} \beta_{w1}}{\operatorname{tg}(\beta_{w1} + \rho)} = \frac{\operatorname{tg}(\lambda_4 - \rho)}{\operatorname{tg} \lambda_4},$$

Where  $\lambda_4 = 90^\circ - \beta_{w1}$  is the lead angle of the worm thread in the initial circumference;

$$\rho = \operatorname{arctg} \frac{f}{\cos \alpha_n}.$$

The conditions for obtaining a self-stopping screw transmission, having the properties of the first self-stopping version, are as follows:





$$\operatorname{ctg}\beta_{w1} < \frac{f}{\cos\alpha_n}; \quad \operatorname{ctg}\beta_{w2} > \frac{f}{\cos\alpha_n}.$$

During the operation in traction mode of forward stroke the coefficient of efficiency is determined with the formulae of a non-self-stopping transmission, and in release mode as follows:

- at  $\sigma = \beta_{w1} - \beta_{w2}$ :

$$\mu_{12} = \frac{f \operatorname{tg}\beta_{w1} - \cos\alpha_n}{\cos\alpha_n - f \operatorname{tg}\beta_{w2}};$$

- at  $\sigma = \beta_{w1} + \beta_{w2}$ :

$$\mu_{12} = \frac{f \operatorname{tg}\beta_{w1} - \cos\alpha_n}{\cos\alpha_n + f \operatorname{tg}\beta_{w2}};$$

- worm gears:

$$\mu_{12} = -\frac{\operatorname{tg}\beta_{w1}}{\operatorname{tg}(\beta_{w1} + \rho)} = \frac{\operatorname{tg}(\rho - \lambda_1)}{\operatorname{tg}\lambda_1}.$$

It should be noted that in the self-stopping worm gear  $\beta_{w1} + \rho > 90^\circ$  and  $\rho > \lambda_1$ .

The second self-stopping alternative, when the screw transmission is able to work only in traction mode of the forward stroke, is only possible in the transmissions with positive values of friction parameters. The conditions for this alternative in accordance with (34) are as follows:

$$\operatorname{tg}\beta_{w1} > \operatorname{tg}\beta_{w2} \geq \frac{\cos\alpha_n}{f}.$$

The coefficient of efficiency of the forward stroke for this alternative is determined by the same formula that was used for the non-self-stopping transmission with positive friction parameters, i.e. at  $\sigma = \beta_{w1} + \beta_{w2}$ .

The third self-stopping alternative, when the screw transmission is able to operate only in brake release mode of the backward stroke lead, is only possible in the transmissions with different signs of friction parameters. The conditions of this alternative in accordance with (35) are as follows:

$$\operatorname{tg}\beta_{w1} \leq -\frac{f}{\cos\alpha_n}; \quad \operatorname{tg}\beta_{w2} < \frac{f}{\cos\alpha_n}.$$

The coefficient of brake release is determined by the formula given above for the transmission of the first self-stopping alternative at  $\sigma = \beta_{w1} + \beta_{w2}$ .

### III. CONCLUSIONS

The dependencies obtained during the analysis of the friction forces influence in the kinematic pairs of three-link mechanism on its operation under different modes, allow us to draw the following conclusions.

1. The relation of the total moment of the friction forces to the moment of the normal forces relative to the rotation axis of the link can be used as the main parameter to determine the influence of the friction forces on the power transmission process from the driving to the driven link for every movable link, which performs a rotary motion, and for the links which execute a linear motion can be used the relation of the total projection of the friction forces to the projection of the normal force on the motion direction of the link. These relations are called parameters of the friction links.
2. All the characteristics of the mechanism, which depends on the presence of friction forces in its kinematic pairs, can be expressed through the parameters of friction of the movable links: Coefficient of efficiency of forward and backward stroke, coefficient of release, conditions of operability and self-stopping in traction mode and brake release are parameters of the self-stopping alternatives. At the same time, to determine all of these characteristics, it takes only to find the parameters of friction of the movable links in traction mode of the forward stroke.

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